

ESTIMATION OF A COST FUNCTION FOR A  
NAVAL AIR REWORK FACILITY

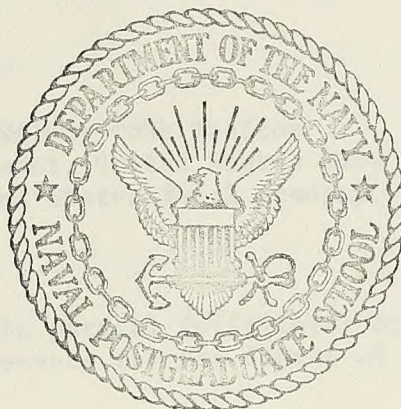
Wilbur Cobb Trafton

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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

ESTIMATION OF A COST FUNCTION FOR A  
NAVAL AIR REWORK FACILITY

by

Wilbur Cobb Trafton

Thesis Advisor:

Norman K. Womer

March 1973

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Estimation of a Cost Function for a  
Naval Air Rework Facility

by

Wilbur Cobb Trafton  
Lieutenant, United States Navy  
B.S., United States Naval Academy, 1966

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## ABSTRACT

The objective of this study was to estimate a cost function from a Constant Elasticity of Substitution production function and a Cobb-Douglas production function for the aircraft rework and engine repair programs at the Naval Air Rework Facility, North Island, San Diego, California. The cost functions were estimated by multiple regression analysis, from data aggregated from actual data taken from production records of the two programs. An attempt was made to validate the two cost functions that were obtained, and a methodology was outlined for comparing predicted costs to actual production costs at the Naval Air Rework Facility.





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## I. INTRODUCTION

### A. ORGANIZATION, NARFNI

The Naval Air Rework Facility, Naval Air Station, North Island, San Diego, California, (NARFNI) is one of seven rework facilities serving the U.S. Navy and the U.S. Marine Corps. It is directly responsible for all major maintenance, incorporation of technical changes, and repair of extensively damaged West Coast based F-4, F-8, C-2, and E-2 aircraft and H-3, H-46, and H-53 helicopters.

Bradley [1] covers the organization and operating procedures at NARFNI in a very detailed manner. What follows is a brief description of NARFNI.

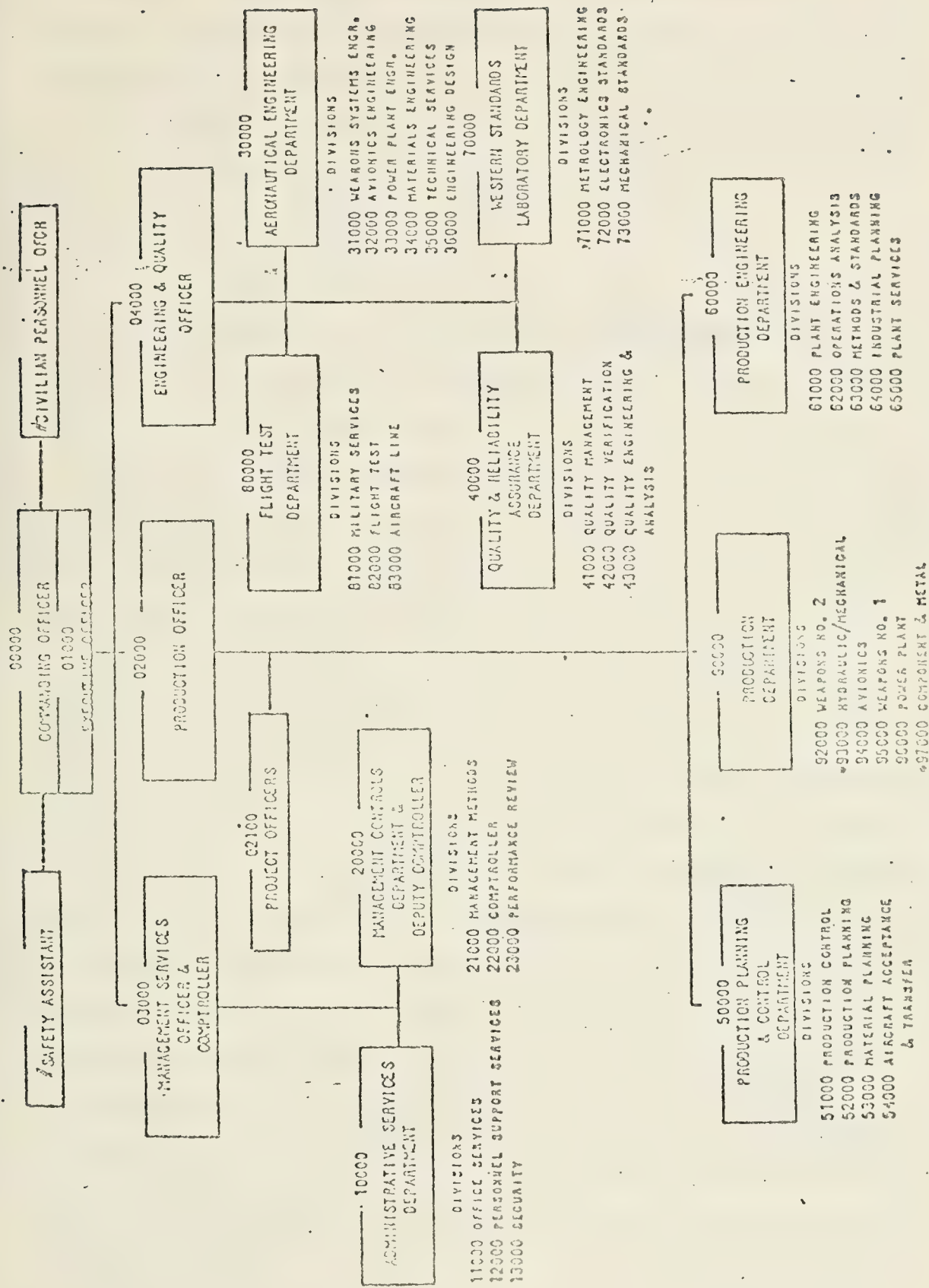
NARFNI is directly responsible to the Naval Air Systems Command Representative, Pacific, who through the Commander, Naval Air Systems Command, is responsible to provide aircraft maintenance to the Commander-in-chief, Pacific Fleet, in the operational chain of command.

NARFNI is composed of nine major divisions (Figure 1), and each division is composed of a direct labor force and an indirect labor force. The direct labor force encompasses all skilled tradesmen, while the indirect labor force encompasses all managerial and administrative personnel.

All aircraft in the Naval Service are scheduled for preventive maintenance on a regular Progressive Aircraft Rework Cycle (PAR Cycle) during their life span. While undergoing PAR, necessary modifications to existing systems are made and major overhaul of component parts is carried out. Prior to induction into a specific PAR Cycle, an estimate, based on historical data of man hours required to rework similar aircraft,







NAVAL AIR REWORK FACILITY, SAN DIEGO, ORGANIZATIONAL CHART

FIGURE 1





is obtained. This estimate is referred to as Production Load Norm (NORM). Managers then plan the rework cycle for each aircraft, ordering material as required and allocating existing manpower.

NARFNI is engaged in three major programs:

1. Airframe Repair
2. Engine Repair
3. Component Repair

Most of the work is for the Naval Air Systems Command Representative, Pacific. Small amounts of work are done for other customers, particularly the Air Force and the Coast Guard.

Most of the work at NARFNI is done on a fixed price contract basis. Costs negotiated in this contract between the customer and NARFNI include direct labor and direct material costs, as well as an overhead cost to cover indirect labor, station support services, and employee benefits.

NARFNI is funded primarily by the Navy Industrial Fund (NIF). Basically the NIF system provides a working capital fund to finance continuing cycles of ongoing operations. Receipts derived from these operations are used to perpetuate the fund. All work done by NARFNI is paid for initially out of the NIF which is then reimbursed from customer appropriations. NARFNI is required by NAVAIRSYSCOM to plan and control its financial affairs in an attempt to incur zero loss at the end of each fiscal year. Prices for services rendered during each quarter are established through negotiations with the customer at the beginning of each quarter during the Quarterly Fleet Readiness Support Conference. It is NARFNI's goal to balance deficits or surpluses in one quarter with compensating surpluses and deficits the next, thus maintaining the NIF at a constant value.



## B. WIPICS

In 1972 NARFNI installed a computerized Work in Process Inventory Control System (WIPICS) developed by the Rohr Corporation, Chula Vista, California. Spooner [2] gives a good overview of the history behind the installation of WIPICS at NARFNI and the details involved in actual day to day operation of WIPICS. The system employs many of the latest advances in computer technology which include real time data file updating and information retrieval with communications via Touch Tone<sup>1</sup> (R) telephones and remote teletypewrite terminals. Basically, the system operates as follows. Each individual item or component is assigned a unique register number which is used for all WIPICS transactions. Each time the status or location of the component changes, a transaction updates the computer. The system provides immediate audio response (pre-recorded voice) for simple questions and printed reports for more detailed questions. A query as to the location or status of a component or part is answered with current information on that item.

## C. PRIOR THESIS WORK

The Management Systems Development Office (MSDO) at North Island and NARFNI have been charged by higher authority with determining the cost-effectiveness of WIPICS as one factor in justification of continuing the use of WIPICS at NARFNI and possible installation of WIPICS at other rework facilities. In conjunction with this, a project is continuing at the Naval Postgraduate School to develop a methodology for auditing cost-effectiveness analysis of major technological changes. This methodology is then to be applied to WIPICS at NARFNI.

---

<sup>1</sup> Touch Tone (R) is a registered trade mark.





Three recent graduates of the Operations Research Curriculum at the Naval Postgraduate School each wrote a thesis as part of the above project. Spooner [2] wrote on "Evaluation of a Technological Change in Production". He outlined the methodology one might use in determining the cost-effectiveness of WIPICS, but he did not actually apply his methodology. In theory, his work takes one from production functions to cost functions to product transformation curves. Bradley [1] used actual production data taken from "pre-WIPICS" production records to estimate a continuous Cobb-Douglas production function for the aircraft rework, engine repair, and component repair programs. Meyers [3] constructed a linear economic model for the aircraft rework and engine repair programs which predicts resource requirements and costs for several alternative objective functions.

#### D. SCOPE OF THIS STUDY

Bradley [1] did not carry his research to the next step, which, following Spooner's methodology, is to estimate a cost function for each program of the NARF. There is also evidence which indicates the existence of certain flaws in using Bradley's estimated Cobb-Douglas production functions to directly construct cost functions. This is covered in more detail in the next chapter.

The purpose of this study is to estimate a cost function for the aircraft rework and engine repair programs at NARFNI through multiple regression analysis of actual production data at NARFNI from March, 1970, to August, 1971. This is the second step in Spooner's three step methodology for determining the cost-effectiveness of WIPICS. Using methods of reduced form estimation, a cost function will be estimated from a Constant Elasticity of Substitution (CES) production function and a



Cobb-Douglas production function. The two cost functions will then be compared to determine which one more accurately predicts total production costs. Finally, a methodology will be outlined for comparing the best cost equation and the Linear Programming (LP) Model to the data and to each other.





## II. THE PROBLEM

### A. APPLICATION OF THE COBB-DOUGLAS MODEL

Spooner [2, p. 33] discusses a computerized prorating program designed to prorate the raw data provided by NARFNI for each engine and aircraft that underwent repair during a specified period. Assuming seven day work weeks and equal distribution of labor hours and material dollars on a daily basis, the program prorates NORM, direct labor dollars, direct labor hours, direct material costs, and overhead costs for each individual engine and aircraft.

Using aggregated prorated data obtained from the prorate program, Bradley [1] estimated Cobb-Douglas production functions for aircraft rework and engine repair programs at NARFNI as follows (the equations are presented in logarithmic form to facilitate display of standard error of each coefficient, shown in parenthesis, and overall  $R^2$ , shown at the right of each equation):

#### Aircraft

$$\ln APH = 4.04 + .38104 \ln APL - .23714 \ln APM + .88208 \ln N \quad R^2 = .920$$

(.044)                      (.018)                      (.044)

#### Engines

$$\ln APH = -1.898 + .92210 \ln APL + .25135 \ln APM \quad R^2 = .981$$

(.009)                      (.011)

APH is aggregate prorated hours of norm, APL is aggregate prorated hours of labor, APM is aggregate prorated material dollars, and N is number of jobs in shop.



From Naval Air Rework Facility Production and Planning Notices dated 17 June 1971, and 26 July 1971, forecast workload requirements for engine and aircraft programs during third quarter fiscal 1972 were obtained. This information was then used in Bradley's models to predict quarterly direct labor requirements and total cost. One result of comparing actual and predicted data appears in Figure 2. Here an isoquant was plotted for the aircraft rework program, holding N constant and plotting labor versus material. An interesting result immediately became evident - all production data points fell in the upper left hand portion of the curve. Furthermore, when the isocost line was plotted for the program (using prices of labor and material from Meyers' thesis), the theoretical point of most efficient operation, which is the point of tangency between the

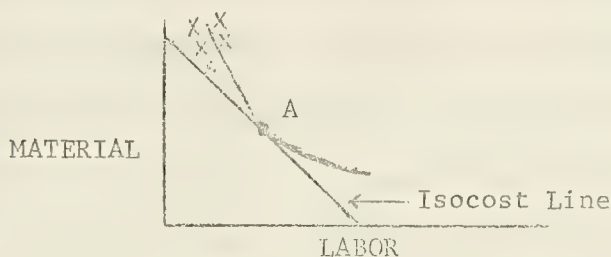


Figure 2. Plot of Cobb-Douglas Model (Aircraft)

isoquant curve and the isocost line (Point A), fell far out of the region of actual operation. This implies one of two things; either NARFNI is operating very inefficiently, or the Cobb-Douglas production function, as Bradley estimated it, is not a good description of the actual process by which managers decide on quantities of inputs to use in the repair programs at NARFNI. Given the number of variables which enter into the production process at NARFNI that are not included in Bradley's model, i.e. managerial decisions and constraints on such items as work flow, allocation of resources, and overtime hours permitted, the latter reason



given above appears to be more realistic than the former. All of the estimation conducted in this study was carried out under the assumption that NARFNI was in fact operating efficiently.

#### B. NEED FOR A COST MODEL

The problem is one of estimating a realistic cost function for NARFNI as the second step in Spooner's three step process for determining the cost-effectiveness of WIPICS. This cost function can then be used to complete the third step, which is formulation of product transformation curves.

Because of the problems encountered in application of Bradley's Cobb-Douglas production functions, it was decided to use methods of reduced form estimation to obtain a cost function and provide estimates of the coefficients in the CES and Cobb-Douglas models. Another reason for not using Bradley's model was the desire to include penalty costs, as determined by Meyers [3], in a cost function as a measure of pipeline cost per unit. Penalty costs are the daily costs to the Navy for having an aircraft or engine in the rework cycle. Penalty costs for aircraft are computed by dividing procurement cost by expected life. Penalty costs for engines are determined by dividing procurement cost by ten years. By including an index of the price of labor (prorated labor costs divided by prorated labor hours) and an index of pipeline costs (aggregated penalty costs divided by daily number of items in shop), predicted costs will be responsive to increases in wages as well as increased sophistication of the items undergoing rework.





### III. DERIVING COST FUNCTIONS

#### A. THEORY

##### 1. General

The theory behind the derivation of cost functions in this chapter is based on three very important assumptions. First, we assume that it is the objective of the NARF to minimize costs. The second assumption is that the NARF has no control over prices of labor or penalty costs per item per day. Third, we assume that output is fixed at any desired level and the NARF does an imperfect job of minimizing costs for a given level of output. The reason for this is that when the NARF makes a decision it doesn't know exactly what the prices will be, and it does not know until an item arrives exactly how extensive a rework will be required. According to Nerlove [4, p. 107], the fundamental duality between costs and production functions assures us that the relation between the cost function, obtained empirically, and the underlying production function is unique.

##### 2. CES Production Function

The general form of the CES production function is as follows [5, p. 284]:

$$Y = \gamma(\delta X_1^{-\rho} + (1-\delta)X_2^{-\rho})^{-\sigma/\rho}$$

where  $\gamma$  is an efficiency parameter,  $\rho$  is a substitution parameter, and  $\delta$  is a distribution parameter.  $\sigma$  is a measure of returns to scale, i.e., if  $\sigma = 1$ , there are constant returns to scale, and if  $\sigma > 1$  or  $\sigma < 1$  there are increasing or decreasing returns to scale respectively.  $Y$  is production output, while  $X_1$  and  $X_2$  are inputs to production.



What follows is the derivation of a cost function from the production function and marginal productivity relations. See Walters [5, p. 284] for a more detailed explanation.

$$\min C = P_1 X_1 + P_2 X_2 \quad (1)$$

$$\text{s.t. } Y = \gamma(\delta X_1^{-\rho} + (1 - \delta)X_2^{-\rho})^{-\sigma/\rho}$$

where  $C$  is production cost,  $P_1$  is the price of input  $X_1$ , and  $P_2$  is the price of input  $X_2$ . Forming the Lagrange Equation and taking partial derivatives,

$$\mathcal{L} = P_1 X_1 + P_2 X_2 - \lambda[Y - \gamma(\delta X_1^{-\rho} + (1 - \delta)X_2^{-\rho})^{-\sigma/\rho}] \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial X_1} = \frac{-\sigma}{\rho} \gamma(\delta X_1^{-\rho} + (1 - \delta)X_2^{-\rho})^{(-\sigma/\rho)-1} (1 - \delta)(-\rho)X_1^{-\rho-1} = \lambda P_1 \quad (3)$$

$$\frac{\partial \mathcal{L}}{\partial X_2} = \frac{-\sigma}{\rho} \gamma(\delta X_1^{-\rho} + (1 - \delta)X_2^{-\rho})^{(-\sigma/\rho)-1} (\delta)(-\rho)X_2^{-\rho-1} = \lambda P_2 \quad (4)$$

Forming the ratio of equations (3) and (4),

$$\frac{P_1}{P_2} = \frac{(1 - \delta)X_1^{-\rho-1}}{\delta X_2^{-\rho-1}} \quad (5)$$

$$\text{Thus, } X_1 = \left[ \frac{\delta P_1}{(1 - \delta) P_2} \right]^{\frac{1}{-\rho-1}} X_2 \mu \quad (6)$$

where  $\mu$  is a random error term due to imperfect minimization of costs.





Now,

$$\frac{X_1}{X_2} = \left[ \frac{P_1}{(1 - \delta) P_2} \right]^{\frac{1}{-\rho - 1}} \mu \quad (7)$$

Taking natural logarithms of both sides of (7).

$$\ln(X_1/X_2) = a + b \ln(P_2/P_1) + \ln \mu \quad (8)$$

where

$$a = \left( \frac{\delta}{1 - \delta} \right)^{\frac{1}{-\rho - 1}} \quad \text{and } b = \frac{1}{-\rho - 1}$$

Substituting back into the cost equation,

$$X_2 = \frac{C}{P_1(X_1/X_2) + P_2} \quad (9)$$

The production function is now

$$Y = \gamma [\delta + (1 - \delta)(X_1/X_2)^{-\rho}] X_2^{-\sigma/\rho} \quad (10)$$

Solving for  $X_2$ ,

$$X_2 = \left[ \frac{\left( \frac{Y}{\gamma} \right)^{-\rho/\sigma}}{\delta + (1 - \delta)(X_1/X_2)^{-\rho}} \right]^{1/(-\rho)} \quad (11)$$

Taking natural logs of both sides of equation (11),

$$\ln X_2 = \left[ -\frac{1}{\sigma} \ln \gamma \right] + \left( \frac{1}{\sigma} \right) \ln Y + \left( \frac{1}{-\rho} \right) \left[ \delta + (1 - \delta)(X_1/X_2)^{-\rho} \right] \quad (12)$$



To simplify handling equation (12), let

$$\ln X_2 = [\alpha] + \beta \ln Y + (\omega)[D] \quad (13)$$

The final form of the cost equation is

$$C = (P_2 + P_1(X_1/X_2))e^{\alpha} Y^{\beta} e^{\omega D} \epsilon \quad (14)$$

where  $\epsilon$  is a random error term.

### 3. Cobb-Douglas Production Function

The general form of the Cobb-Douglas production function is

[5, p. 275]:

$$Y = AX_1^{\alpha} X_2^{\beta}$$

where  $Y$  is production output,  $X_1$  and  $X_2$  are production inputs, and  $\alpha$  and  $\beta$  are elasticities of production with respect to  $X_1$  and  $X_2$  respectively.

Once again the specification of the production function is changed so that  $Y$  is non-random, but  $X_1$  and  $X_2$  are random variables, due to the fact that the NARFNI does an imperfect job of minimizing costs for a given level of output and has no control over prices. With this in mind, a cost function is derived as follows:

$$\min C = P_1 X_1 + P_2 X_2 \quad (15)$$

$$\text{s.t. } Y = AX_1^{\alpha} X_2^{\beta}$$

Forming the Lagrangian Equation and taking partial derivatives,

$$\mathcal{L} = P_1 X_1 + P_2 X_2 - \lambda(Y - AX_1^{\alpha} X_2^{\beta}) \quad (16)$$



$$\frac{\partial f}{\partial X_1} = P_1 - \lambda \alpha A X_1^{\alpha-1} X_2^\beta \mu_1 = 0 \quad (17)$$

$$\frac{\partial f}{\partial X_2} = P_2 - \lambda \beta A X_1^\alpha X_2^{\beta-1} \mu_2 = 0 \quad (18)$$

$$\frac{\partial f}{\partial \lambda} = Y - A X_1^\alpha X_2^\beta e^\mu = 0 \quad (19)$$

where the  $\mu$ 's are random variables associated with error (again, due to imperfect minimization of costs). Taking the ratio of equations (17) and (18),

$$\frac{\alpha X_2}{\beta X_1} = \frac{P_1}{P_2} \mu \quad (20)$$

Solving for inputs  $X_1$  and  $X_2$ , the following cost function is obtained:

$$C = \left[ \frac{Y}{A} \left( \frac{\alpha}{\beta} + 1 \right)^\beta \left( \frac{\beta}{\alpha} + 1 \right)^\alpha P_1^\alpha P_2^\beta \right]^{1/(\alpha+\beta)} \mu - \frac{\beta}{\alpha + \beta} \left( \frac{\beta \mu}{\alpha} + 1 \right) \quad (21)$$

## B. PRODUCT TRANSFORMATION CURVES

Spooner [2, Appendix E] contains a complete theoretical derivation of product transformation curves from cost functions. Basically, a product transformation curve is obtained by rewriting a cost function in terms of output. The result of the substitution is a nonlinear relationship between two or more outputs. By looking at two outputs at a time, setting the others equal to a constant, and setting cost equal to





a constant, graphical analysis can be applied to the overall cost function. After plotting one output against the other, the shape of the product transformation curve will indicate increasing, constant, or decreasing returns to scale (see Figure 3).

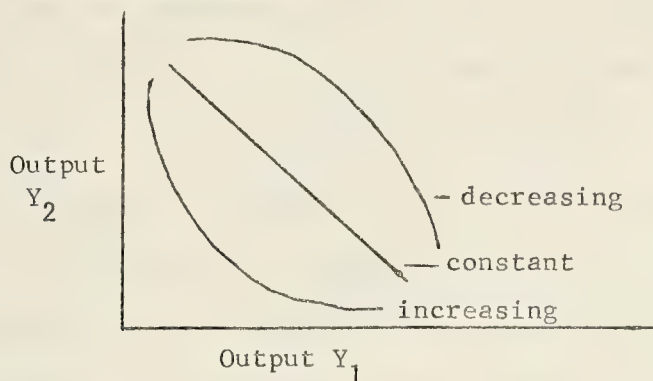


Figure 3. Product Transformation Curves, Returns to Scale

Product transformation curves may also be used as one way of comparing the effects of WIPICS on production at NARFNI. A comparison could be made between before WIPICS and after WIPICS transformation curves, keeping the total budget constant and keeping prices of resources and inputs the same for the two periods. One should be very careful in evaluating the results of such a comparison because of the many factors which enter into production at NARFNI but do not appear in a cost function, particularly management decisions.



#### IV. ESTIMATING COST FUNCTIONS

##### A. PRORATING WITH PENALTY COSTS

The raw data on each aircraft and engine job completed during the period February, 1970 to August, 1971, which appears in Meyers' thesis [3, Appendix A] was run through the computer prorate program in Spooner's thesis [2, Appendix B] in order to provide daily aggregated, prorated data which could be used in a linear regression. In addition, penalty costs for each type of engine and aircraft (see Table I and Table II) were incorporated into Spooner's prorate computer program so that these costs could be used in estimating a cost function for NARFNI. The codes shown in Tables I and II are as defined by Meyers [3, p. 16]. The method used to incorporate these costs into the program was simply to read the aircraft or engine code from the raw data card for all items that were in shop each day, and then the appropriate daily penalty costs were aggregated for each day of the period under observation. The aggregated prorated data used to estimate the cost functions in this thesis appear in Appendix A (aircraft) and Appendix B (engines). Spooner's prorate programs, rewritten to include penalty costs, appear in Appendix D (aircraft) and Appendix E (engine).

The interest in penalty costs stems from the fact that this appeared to be the only way to quantify unit pipeline costs, and it was considered desirable to include pipeline costs in any cost model of NARFNI.



TABLE I. DAILY ENGINE PENALTY COSTS

<u>CODE</u>	<u>TYPE ENGINE</u>	<u>PENALTY COST</u>
51	T58-GE-8B	17.81
53	T58-GE-8B	17.81
55	T58-GE-8B	17.81
57	T58-GE-8B	17.81
81	T58-GE-8B	17.81
82	T58-GE-8B	17.81
83	T58-GE-8B	17.81
84	T58-GE-8B	17.81
85	T58-GE-8F	17.81
86	T58-GE-8F	17.81
87	T58-GE-8F	17.81
89	T58-GE-8F	17.81
63	T64-GE-6B	34.25
65	T64-GE-6B	34.25
66	T64-GE-6B	34.25
67	T64-GE-6B	34.25
69	T64-GE-413	45.25
56	T56 ALL MODELS	27.01
71	J57 ALL MODELS	57.53
72	J57 ALL MODELS	57.53
73	J57 ALL MODELS	57.53
74	J57 ALL MODELS	57.53
75	J57 ALL MODELS	57.53
91	J79-GE-8B	61.64
92	J79-GE-8C	61.64
95	J79-GE-10	49.39





TABLE II DAILY AIRCRAFT PENALTY COSTS

<u>CODE</u>	<u>TYPE AIRCRAFT</u>	<u>PENALTY COST</u>
10	C-2A	280.29
11	E-2B	280.29
21	F-4J	85.34
22	F-4B	94.38
23	RF-4B	77.02
25	F-8J	66.05
26	F-8H	68.59
27	RF-8G	47.48
31	CH-3B	43.76
32	RH-3A	51.64
33	SH-3A	43.76
34	SH-3A/G	36.43
35	SH-3D	36.43
41	CH-46A	37.08
42	CH-46D	37.08
43	CH-46F	37.08
44	UH-46A	28.56
45	UH-46D	28.56
48	CH-53A	41.46
49	CH-53D	41.46



## B. REGRESSION PROCEEDURES AND RESULTS

### 1. CES Proceedures

For the CES production function a cost function was estimated as follows (all values used were taken from the prorated data, cut on each end to eliminate start up and shut down effects as per Spooner [2, p. 33]):

$$\min C = PI (I) + PL (L)$$

$$\text{s.t. } N = \gamma(\delta L^{-\rho} + (1 - \delta)I^{-\rho})^{-\sigma/\rho}$$

where  $PI = \frac{\text{penalty cost}}{\# \text{ of items in shop}}$

$$PL = \frac{\text{direct labor cost}}{\text{direct man hours}}$$

$$I = \text{number of items in shop}$$

$$L = \text{direct man hours}$$

$$N = \text{NORM}$$

Following the procedure as set forth in the previous chapter, equations (1) through (8), the first stepwise linear regression was run on equation (8) of Chapter 3. This equation is listed below, with  $\ln \mu$  representing the natural log of the error term:

$$\ln(I/L) = a + b \ln(PL/PI) + \ln \mu \quad (1)$$

where  $a = \left( \frac{\delta}{1 - \delta} \right)^{\frac{1}{-\rho - 1}}$  and  $b = \frac{1}{\rho + 1}$ . The error was assumed to be normally distributed with a mean of zero and constant variance.



The regression on equation (1) provided an estimate of  $(I/L)$ , which will be indicated as  $(\hat{I}/L)$ , for use in the following equation:

$$L^* = \frac{C}{PI(\hat{I}/L) + PL} \quad (2)$$

which corresponds to equation (9) in the previous Chapter. Here  $C$  equals direct labor cost plus overhead cost plus penalty cost. The regression on equation (1) also provided the parameters for obtaining the quantity  $\hat{D}$  in equation (4) below. The final equation on which a second stepwise linear regression was run was (see equation (12), Chapter III):

$$\ln L^* = -\left[\frac{1}{\sigma} \ln \gamma\right] + \left(\frac{1}{\sigma}\right) \ln N + \left(\frac{1}{-\rho}\right) \left[\delta + (1 - \delta)(\hat{I}/L)^{-\rho}\right] + \epsilon \quad (3)$$

where  $\epsilon$  is a random error term. Simplifying the quantities in equation (3), let

$$\ln L^* = a + b \ln N + C[\hat{D}] + \epsilon \quad (4)$$

The second regression provided estimates of the coefficients of the independent variables.

All regressions in this thesis were run using the SNAP/IEDA statistical package available at the Naval Postgraduate School Computer Center.

## 2. CES Regression Results

The results of the first CES regression for the aircraft and engine programs are presented in Tables III and IV respectively. The low  $R^2$  (.05) in the aircraft regression is an indication of possible problems in applying the CES production function to the production process at NARFNI ( $R^2$  is the square of the multiple correlation between the dependent variable and those independent variables which were included in the





TABLE III. FIRST CES STEPWISE REGRESSION OF AIRCRAFT DATA

<u>STEP</u>	<u>1</u>
Entered	PL/PI
Previously Entered	-----
$R^2$	.054
Standard Error of Dependent Var.	.036
Coefficients	
Constant	-4.010
PI/PL	.233
Standard Error of Coefficients	
PI/PL	.058
CV	-.0078



TABLE IV. FIRST CES STEPWISE REGRESSION OF ENGINE DATA

<u>STEP</u>	<u>1</u>
Entered	PL/PI
Previously entered	----
$R^2$	.440
Standard error of dependent var.	.074
Coefficients	
Constant	-0.219
PI/PL	1.324
Standard Error of Coefficients	
PI/PL	.079
CV	- .0266



regression at that step -- it measures the proportion of variation in the dependent variable that is explained by the independent variable). Rao and Miller [6, p. 16] point out that a high  $R^2$  may imply the appropriateness of a regression equation for explaining the movements of a dependent variable, but a low  $R^2$  does not necessarily imply that the regression equation is inappropriate. Another possible problem appeared when the value of  $\delta$  was computed. It is specified by Walters [5, p. 284] that  $0 \leq \delta \leq 1$  in a CES production function. This means that the expected value of the constant in the first regression runs was a positive number less than one. This was not the case for either program. As a matter of interest, the coefficient of variation (CV), which is standard error divided by mean of the dependent variable, is listed in all tabled regression results in this Chapter. It is a measure of dispersion of the dependent variable in comparison to the mean due to lack of a perfect fit with the data.

Looking at the results of the second regression of the aircraft data (Table V), the extremely high  $R^2$  (.988) indicates a good fit with the data. Both independent variables, N and D, were found to have significant coefficients after examination of their respective standard errors (when computation of the standard deviation is based on the estimate of the variance rather than the variance itself, it is called standard error).

In the engine program (Table VI), both independent variables were again found to have coefficients significantly different from zero.  $R^2$  in this case (.798) was not nearly as high as it was for the aircraft program, but it is certainly acceptable.

The value of  $\rho$ , computed from the coefficient of D in each case (the coefficient of D equals  $-\frac{1}{\rho}$ ), shows  $\rho$  to fall well within the





TABLE V SECOND CES STEPWISE REGRESSION OF AIRCRAFT DATA

<u>STEP</u>	1	2
Entered	N	D
Previously entered		N
$R^2$	.977	.988
Standard error of dependent variable	.012	.009
Coefficients		
Constant	1.596	2.063
N	.897	.880
D		-2.388
Standard Error of Coefficients		
N	.008	.006
D		.149
CV		.0006



TABLE VI SECOND CES REGRESSION OF ENGINE DATA

<u>STEP</u>	1	2
Entered	N	D
Previously entered		N
$R^2$	.775	.798
Standard error of dependent variable	.066	.062
Coefficients		
Constant	.753	2.344
N	.983	.929
D		4.566
Standard Error of Coefficients		
N	.028	.028
D		.721
CV		.0075



expected region. This value was .419 for the aircraft program and -.219 for the engine program. The expected value of  $\rho$  is a number between minus one and infinity. As  $\rho$  approaches minus one the result is a flat isoquant, and as  $\rho$  increases in a positive direction the elasticity of substitution becomes smaller and smaller (essentially this is the case of fixed coefficients). For a more detailed explanation of this, see Walters [5, p. 286].

The final estimated cost functions were:

$$\text{(aircraft)} \quad C = (PL + PI(\hat{I}/L))e^{2.063N^{.880}e^{-2.388(\hat{D})}} \quad (5)$$

$$\text{(engines)} \quad C = (PL + PI(\hat{I}/L))e^{2.344N^{.929}e^{4.566(\hat{D})}} \quad (6)$$

### 3. Cobb-Douglas Procedures

For the Cobb-Douglas production function, a cost function was estimated using the general theory presented in equations (15) through (21) in Chapter III. The specific equations used in the estimation procedure were:

$$\min C = PL(L) + M + PI(I)$$

$$\text{s.t. } N = AL^{\alpha}I^{\beta}M^{\gamma}$$

All variables are as previously defined, and M is material costs (again, prorated data was used in all cases). A stepwise linear regression was run on the following equation:

$$\begin{aligned} \log C = a + \frac{1}{\alpha + \beta + \gamma} \log N + \frac{\alpha}{\alpha + \beta + \gamma} \log PL \\ + \frac{\beta}{\alpha + \beta + \gamma} \log PI + \epsilon \end{aligned} \quad (7)$$





where  $a = \log\left(\frac{\alpha + \beta + \gamma}{\gamma}\right) - \frac{1}{\alpha + \beta + \gamma} \log A$

$$+ \frac{\alpha}{\alpha + \beta + \gamma} \log \left(\frac{\gamma}{\alpha}\right) + \frac{\beta}{\alpha + \beta + \gamma} \log \left(\frac{\gamma}{\beta}\right) .$$

Again, the error term is assumed to be normally distributed with a mean of zero and constant variance. The regression provided estimates of the coefficients of N, PL, and PI.

#### 4. Cobb-Douglas Regression Results

The results of the regressions run on equation (7) are presented in Tables VII and VIII for the aircraft and engines programs, respectively. Examination of this tabulated data provides a good indication that the cost functions estimated from the Cobb-Douglas production function may turn out to be valid and accurate cost functions.

In the aircraft data regression, all coefficients were significant, and the value of  $R^2$  (.960) was an indication of a good fit with the data. The regression on the engine data showed only NORM to have a significant coefficient. The coefficient of PL(-.027) had a standard error of .113, and the coefficient of PI(.101) had a standard error of .051, indicating that neither coefficient was significantly different from zero.  $R^2$  for the engine data was .848.

The resulting cost functions for the aircraft and engine programs were:

$$(\text{aircraft}) \quad C = 10^{.421} N^{.751} PL^{2.629} PI^{-.275} \quad (8)$$

$$(\text{engines}) \quad C = 10^{1.749} N^{.860} \quad (9)$$



TABLE VII COBB-DOUGLAS STEPWISE REGRESSION OF AIRCRAFT DATA

<u>STEP</u>	<u>1</u>	<u>2</u>	<u>3</u>
Entered.	N	PL	PI
Previously entered		N	N
			PL
$R^2$	.645	.949	.960
Standard error of dep. var.	.024	.009	.008
Coefficients			
Constant	1.678	-0.049	0.421
N	0.824	0.769	0.751
PL		2.453	2.629
PI			-0.275
Standard error of Coefficients			
N	.036	.014	.012
PL		.060	.056
PI			.030
CV	.0049	.0018	.0016



TABLE VIII COBB-DOUGLAS STEPWISE REGRESSION OF ENGINE DATA

<u>STEP</u>	1	2	3
Entered	N	PI	PL
Previously entered		N	PI
			N
$R^2$	.848	.849	.849
Standard error of dep. var.	.020	.020	.020
Coefficients			
Constant	1.749	1.539	1.554
N	.860	.876	.876
PI		.097	.101
PL			-.027
Standard error of coefficients			
N	.019	.021	.021
PI		.047	.051
PL			.113
CV	.0044	.0044	.0044



## V. VALIDATION

### A. APPLICATION OF THE COST MODELS

The prorated data was broken down into nine consecutive 30 day intervals beginning with julian date 0191 and ending with julian date 1095. Average daily values of NORM, total costs (labor plus material plus penalty), PL, and PI were computed for each period. This was done for both the aircraft and engine programs (see Appendix C).

For validation of the CES cost functions, the values of NORM, PL, PI,  $\hat{I}/L$ , and  $\hat{D}$  were computed for the nine 30 day periods and then used in equations (5) and (6) in Chapter IV. The results of these runs, shown in Table IX, are evidence that the CES cost function can in fact be used to predict costs at NARFNI (remembering that actual costs shown here are average daily prorated direct labor cost plus overhead cost plus penalty cost). The worst prediction occurs in the first engine prediction, where the difference between predicted and actual cost is only approximately four thousand dollars out of thirty eight thousand, or less than 10%. The percent of error for each program as shown in Tables IX and X was computed by subtracting actual cost from predicted cost and dividing by actual cost. The same independent variables used above, less  $\hat{I}/L$  and  $\hat{D}$ , were then fed into the estimated Cobb-Douglas cost functions which appear at the end of Chapter IV (equations (8) and (9)). Actual and predicted costs for the aircraft and engine programs are listed in Table X for each of the nine 30 day periods. An examination of the data in Table X shows the Cobb-Douglas cost functions to also be predicting costs with fairly good accuracy (again, a reminder that actual costs here are the sum of direct labor, material, and penalty costs).





TABLE IX ACTUAL AND PREDICTED TOTAL COSTS (CES)

AIRCRAFT

<u>PREDICTED</u>	<u>ACTUAL</u>	<u>% ERROR</u>
96,896.19	98,561.24	- 1.69
116,596.25	117,882.93	- 1.09
113,573.44	112,113.25	+ 1.30
116,616.94	115,304.52	+ 1.14
119,525.25	118,598.68	+ .78
110,645.50	109,015.75	+ 1.50
113,536.06	113,658.38	- .08
121,063.69	122,274.12	- .99
114,884.63	115,261.18	- .33

ENGINES

<u>PREDICTED</u>	<u>ACTUAL</u>	<u>% ERROR</u>
34,439.70	38,230.57	- 9.93
36,684.21	39,975.95	- 8.25
30,655.29	32,061.61	- 4.38
36,398.64	36,459.35	- .17
36,644.64	36,471.29	+ .48
29,525.21	30,294.42	- 2.55
35,207.98	34,942.32	+ .76
33,748.04	32,864.77	+ 2.70
29,846.05	29,806.54	+ .13



TABLE X ACTUAL AND PREDICTED TOTAL COSTS (COBB-DOUGLAS)

AIRCRAFT

<u>PREDICTED</u>	<u>ACTUAL</u>	<u>% ERROR</u>
67,304.50	68,452.51	- 1.68
79,858.06	80,122.31	- .33
77,338.25	74,926.64	+ 3.26
78,968.56	78,632.10	+ .43
81,799.38	82,816.17	- 1.23
77,561.00	77,510.11	+ .06
82,942.81	82,667.20	+ .33
89,536.81	89,337.36	+ .22
84,495.63	85,059.84	- .67

ENGINES

<u>PREDICTED</u>	<u>ACTUAL</u>	<u>% ERROR</u>
41,720.43	45,946.33	- 9.20
44,015.41	45,183.24	- 2.59
37,536.84	36,286.71	+ 3.45
43,974.30	41,814.51	+ 5.16
43,680.50	41,564.47	+ 5.09
34,941.77	33,638.37	+ 3.90
41,488.91	41,437.46	+ .12
39,378.77	39,130.91	+ .63
35,189.98	36,008.00	- 2.27



## B. PREDICTION INTERVALS

A prediction interval can be obtained for the Cobb-Douglas aircraft cost function and the engine cost function using methods outlined in Theil [7, p. 135]. This method proceeds as follows. First, return the cost equation to logarithmic form. Then let,

$$Y^* = W'C + \epsilon^*$$

where  $Y^*$  is a single observation of cost in log form,  $W'$  is a one row vector, with each column containing given values of the independent variables,  $C$  is a one column vector with each row being a coefficient of one independent variable, and  $\epsilon^*$  is the random error term with a mean of zero. Let  $Z$  be a matrix of observations on which the regression was run. The first column is all ones, and the remaining columns are data columns. There are as many rows as there are numbers of observations, i.e.,

$$Z = \begin{bmatrix} 1 & N_1 & PL_1 & PI_1 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ 1 & N_N & PL_N & PI_N \end{bmatrix}$$

Now  $\frac{W'C - Y^*}{\sqrt{S^2(1 + W'(Z'Z)^{-1}W)}}$  has a student's  $t$  distribution with  $n-k$  degrees of freedom. Here  $n$  equals number of observations and  $k$  equals number of independent variables in the cost equation.  $S^2$  is an estimate of variance. Selecting a  $1 - \alpha$  confidence interval,

$$P[-t_{\alpha/2} \leq t_{(n-k)} \leq t_{\alpha/2}] = 1 - \alpha$$





Let

$$V = t_{\alpha/2} \cdot S \cdot \sqrt{1 + W'(Z'Z)^{-1}W} \quad , \text{ so that}$$

$$P[W'C - V \leq Y^* \leq W'C + V] = 1 - \alpha$$

Taking antilogs,

$$[10^{(W'C - V)} \leq C \leq 10^{(W'C + V)}]$$

is a  $1 - \alpha$  prediction interval for cost.

### C. METHODOLOGY FOR COMPARISON WITH LP MODEL

A good way to determine the validity of an estimated cost function would be to compare actual costs with estimated costs obtained from the cost function and estimated costs obtained from Meyers [3] LP model. To make the comparison realistic, several important factors must be considered. First, the objective function of the LP model must be to minimize costs. Second, the values of NORM and PL, as well as penalty costs and material costs used in the LP model, must also be used in the cost function. In the LP model, the actual values of NORM for the period of time under observation should be used in the production vector Y. This constrains the model to minimize costs subject to actual values of NORM. The activity vector Z need not be changed, but prices of labor and material used in the resource vector R must coincide with the prices used in the cost function. There remains the task of breaking the data down by type of aircraft or engine and the type of work done on that item for use in the activity vector Z.

Ideally, the two methods of cost prediction should each be run over several periods of time, either 30 day intervals, or monthly, or perhaps



quarterly. This provides enough data points to carry out a meaningful comparison of the two models.

The criteria for determining which cost model is better at predicting costs, as outlined below, is sketchy at best. The lack of independence between the time periods eliminates most conventional statistical tests which might apply. Naturally, if one model is consistently and obviously better than the other, no test need be applied. But if one model is better than the other at times, and then at times worse than the other, the criteria outlined here may aid in determining a "best" model.

The criteria is simply this; take the difference between actual and predicted costs for each time period and square this number. Then sum the squared numbers over the periods of time. Doing this for each model will provide a "mean squared error" (MSE) term for the models,

$$MSE = \frac{\sum_{i=1}^n (C_{pi} - C_{Ai})^2}{n}$$

where  $n$  = number of periods

$C_{pi}$  = predicted cost for period  $i$

$C_{Ai}$  = actual cost for period  $i$

The model with the smallest MSE should be considered the best cost model. Naturally, if the two values of MSE are very nearly the same, there remains a problem of determining whether or not they are significantly different. This problem will not be addressed in this thesis.

As a matter of interest, the values of MSE for the CES and Cobb- Douglas cost functions estimated in Chapter IV of this thesis were as follows:



CES aircraft	<u>1,486,812</u>
CES engines	<u>3,180,710</u>
Cobb-Douglas aircraft	<u>1,083,037</u>
Cobb-Douglas engines	<u>3,584,340</u>



## VI. CONCLUSIONS AND FUTURE STUDY

### A. CONCLUSIONS

The cost functions estimated from the CES production function were:

$$\text{(aircraft)} \quad C = (PL + PI(I/\hat{L}))e^{2.063}N^{.880}e^{-2.388(\hat{D})}$$

$$\text{(engines)} \quad C = (PL + PI(I/\hat{L}))e^{2.344}N^{.929}e^{4.566(\hat{D})}$$

These cost functions were concluded to be a valid means of predicting costs at NARFNI after several validation runs showed predicted and actual average prorated daily costs to compare favorably.

The cost functions estimated from the Cobb-Douglas production function were:

$$\text{(aircraft)} \quad C = 10^{.421}N^{.751}PL^{2.629}PI^{-.275}$$

$$\text{(engines)} \quad C = 10^{1.749}N^{.860}$$

The coefficients of PL and PI were not significant in the engine program. A possible explanation for the relatively high coefficient of PL and the negative coefficient of PI in the aircraft program is that as the cost per unit of items being repaired increases, penalty costs increase, and due to the complexity of the more expensive items, number in shop decreases. Also due to the complexity of more expensive items being repaired, the cost of labor increases rapidly as more skilled tradesman are required.

The cost functions derived from the Cobb-Douglas production function were also concluded to be a valid means of predicting costs at NARFNI.





Predicted and actual average daily prorated costs compared favorably when examined over nine 30 day intervals.

Based on the methodology outlined in this thesis for comparing cost prediction models, the cost functions estimated in this thesis predict costs with approximately the same accuracy.

## B. FUTURE STUDY

A prime area for future study would be to compare the LP model, the Cobb-Douglas cost equation estimated in this thesis, and actual total production costs at NARFNI for several selected periods of time, using the methodology outlined in Chapter IV of this thesis. This would provide a more firm idea of just how accurate and useful the Cobb-Douglas cost function really is.

Another area that could be improved is the method used to determine penalty costs. Rather than simply dividing procurement cost by an expected life for each type aircraft or engine, a more realistic method to use could be as follows:

$$\text{Penalty Cost} = \frac{\text{Cost of all a/c, allowing rotatable spares}}{\text{Cost of all a/c, without rotatable spares}}$$

In other words, the true cost to the Navy for having an aircraft at NARFNI is the cost of an aircraft that had to go to the fleet to replace it (rotatable spares). Dividing this figure by the average length of time an aircraft or engine is undergoing rework would provide a daily penalty cost.

Finally, some method of including management decisions and constraints in a production function must be devised. New equipment, improved procedures, increased worker efficiency as a result of labor force cuts,



hiring and firing policies, and constraints on such items as overtime hours permitted, all enter into the production process at NARFNI, but finding a method to quantify these areas and incorporate them into a production function is a most difficult problem.



APPENDIX A

SUMMARY OF DAILY STATISTICS OF NARF (AIRCRAFT)

DATA FOR JULIAN DATES 0069 TO 1235

TOTAL NUMBER OF OBSERVATIONS = 365

DAY	JUL DATE	NORM	DIR MH	DIR	LAB\$	DIR	MTL\$	AFC	MH	OVHD\$	PEN\$	# IN SHOP
1	69	140.60	146.49	873.54	170.13	24.77	1000.99	94.38	1			
2	70	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
3	71	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
4	72	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
5	73	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
6	74	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
7	75	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
8	76	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
9	77	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
10	78	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
11	79	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
12	80	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
13	81	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
14	82	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
15	83	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
16	84	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
17	85	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
18	86	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
19	87	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
20	88	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
21	89	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
22	90	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
23	91	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
24	92	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
25	93	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
26	94	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
27	95	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
28	96	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
29	97	293.47	303.53	1808.26	358.81	52.91	2073.93	188.76	2			
30	98	458.96	463.45	277.40	62.19	67.62	2722.75	174.10	3			
31	99	458.96	463.45	277.40	62.19	67.62	2722.75	174.10	4			
32	100	458.96	463.45	277.40	62.19	67.62	2722.75	174.10	4			
33	101	458.96	463.45	277.40	62.19	67.62	2722.75	174.10	4			
34	102	458.96	463.45	277.40	62.19	67.62	2722.75	174.10	4			
35	103	458.96	463.45	277.40	62.19	67.62	2722.75	174.10	4			









[illegible]













[illegible]





776	777	778	779	780	781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	796	797	798	799	800	801	802	803	804	805	806	807	808	809	810	811	812	813	814	815	816	817	818	819	820	821	822	823	824	825	826	827	828	829	830	831	832	833	834	835	836	837	838	839	840	841	842	843	844	845	846	847	848	849	850	851	852	853	854	855	856	857	858	859	860	861	862	863	864	865	866	867	868	869	870	871	872	873	874	875	876	877	878	879	880	881	882	883	884	885	886	887	888	889	890	891	892	893	894	895	896	897	898	899	900
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100																									























[illegible]



## SUMMARY OF DAILY STATISTICS OF NARF (ENGINES)

DATA FOR JULIAN DATES 0140 TO 1230

TOTAL NUMBER OF OBSERVATIONS = 1532

DAY	JUL DATE	NORM	DIR	MH	DIR	LAB\$	DIR	MTL\$	OVHD\$	PENALTY\$	# IN SHOP
1	140	10.24	15.40	36	89.60	234.00	174.00	71	102.98	57.33	1
2	141	32.74	40.40	36	234.00	300.00	300.00	09	276.28	118.97	2
3	142	32.74	40.40	36	234.00	300.00	300.00	09	276.28	118.97	2
4	143	32.74	40.40	36	234.00	300.00	300.00	09	276.28	118.97	2
5	144	32.74	40.40	36	234.00	300.00	300.00	09	276.28	118.97	2
6	145	62.25	71.09	59	418.95	636.00	996.00	24	493.02	220.33	5
7	146	97.00	132.22	55	773.60	1019.10	1161.73	34	759.34	431.78	6
8	147	17.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
9	148	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
10	149	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
11	150	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
12	151	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
13	152	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
14	153	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
15	154	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
16	155	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
17	156	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
18	157	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
19	158	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
20	159	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
21	160	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
22	161	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
23	162	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
24	163	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
25	164	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
26	165	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
27	166	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
28	167	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
29	168	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
30	169	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
31	170	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
32	171	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
33	172	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
34	173	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8
35	174	18.18	117.75	45	1019.10	1019.10	1244.77	34	920.76	591.78	8





15. 1122255. 29 25. 50755 41 175 36  
1122255. 45 43 555 177 176 37  
1122255. 34 33 515 178 180 38  
1122255. 40 33 515 179 181 39  
1122255. 42 33 515 180 182 40  
1122255. 22 33 515 181 183 41  
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1122255. 22 33 515 219 221 79  
1122255. 22 33 515 220 222 80  
1122255. 22 33 515 221 223 81  
1122255. 22 33 515 222 224 82  
1122255. 22 33 515 223 225 83













































# APPENDIX C

## AVERAGE DAILY AGGREGATED DATA FOR NINE 30 DAY PERIODS

### NORM

AIRCRAFT	ENGINES
7143.99	2181.62
8735.46	2321.78
8400.21	1929.38
8602.12	2319.26
8746.54	2301.25
7874.49	1775.17
8071.96	2167.55
8637.05	2039.90
8160.86	1789.84

### PL

AIRCRAFT	ENGINES
5.99	5.92
6.05	5.97
6.07	5.87
6.09	5.87
6.15	6.02
6.23	6.15
6.31	6.15
6.36	6.25
6.33	6.13

### PI

AIRCRAFT	ENGINES
86.75	40.58
87.33	40.50
91.01	41.07
92.89	41.17
93.93	40.68
96.81	42.08
91.69	40.76
90.06	40.97
91.26	42.84



## AIRCRAFT PRORATE PROGRAM

69



```

C
DO 70 K=1,20
IF (IAC.EQ.IMATR(K)) GO TO 50
GO TO 60
PCOST=PMTR(K)
50 CONTINUE
60 CONTINUE
70 CONTINUE
DO 1000 J=JDAYS,KDAYS,1
ABOX(J)=ABOX(J)+PNORM
BBOX(J)=BBOX(J)+PMH
CBOX(J)=CBOX(J)+PLAB
DBOX(J)=DBOX(J)+PLAFH
SBOX(J)=SBOX(J)+PKCHD
TBOX(J)=TBOX(J)+PCOST
EBOX(J)=EBOX(J)+1.0
1000 CONTINUE
C
PRINT COMPUTED DATA
C
9999 WRITE (6,9500) JSTART,JSTOP,NCARD
9500 FORMAT (1,/,/,/,18X,'SUMMARY OF DAILY STATISTICS OF NARF',
1, ( AIRCRAFT ),/,/,27X,
2, DATA FOR JULIAN DATES,/,14, TO,/,14,/,27X,JULIAN,15X,
3, TOTAL NUMBER OF OBSERVATIONS =,14,/,/,7X,
4, DIRECT,/,4X, DIRECT,/,5X, DIRECT,/,7X, AFC,/,22X, PENALTY,5X,
5, # IN,/,1X, DAY,/,4X, DATE,/,8X, NGR,/,4X, MANHOURS,4X,
6, LAB COST,/,3X, MATL CGST,/,4X, MANHOURS,3X, OVHD COST,8X,
7, CGST,5X, SHOP,/)
K=JSTOP-JSTART
IF (K.GE.635) K=K-634
LDAYS=JSTART
DO 2000 I=1,K,1
1, LDAYS, ABOX(I), BBOX(I), CBOX(I), DBOX(I), EBOX(I),
6 SBOX(I), TBOX(I), EBOX(I)
9501 FORMAT (14,4X,14,7F12.2,5X,F4.0)
LDAYS=LDAYS+1
IF (LDAYS.EQ.0366) LDAYS=1001
2000 CONTINUE
WRITE (6,9508) ERROR
9508 FORMAT (10X,'NUMBER OF ERRORS =',F3.0)
STOP
END

```



## ENGINE PRORATE PROGRAM

```

DIMENSION PMTR(28),IMATR(28)
DIMENSION ABGX(700),BBGX(700),CBOX(700),DBOX(700),EBOX(700)
DIMENSION FRGX(700),SBOX(700),TBOX(700)
DATA PMTR,IMATR/28*0.0,28*0/
DATA ABGX,BBGX,CBOX,DBOX,EBOX,FRGX,SBOX,TBOX/700*0.0,700*0.0,
6700*0.0,700*0.0,700*0.0,700*0.0,700*0.0,700*0.0/
DATA LK,NOCUR,ERROR/1.0,0.0,0/

CC      READ IN PENALTY COSTS
CC
5000    READ (5,5000) (PMTR(J),J=1,28)
CC      FORMAT (10F7.2)
CC
CC      READ IN ENGINE CODES
CC
5001    READ (5,5001) (IMATR(K),K=1,28)
CC      FORMAT (28I2)
CC
CC      READ IN FIRST AND LAST JULIAN DATE, AND NUMBER OF CARDS
CC
9000    READ (5,9000) JSTART,JSTOP,NCARD
CC      FORMAT (2X,14,2X,14,2X,14)
CC
CC      READ IN RAW DATA
CC
9001    DO 1000 I=1,NCARD,1
CC      READ (5,9001,END=9999) IAC,IN,ICUT,NORM,MH,LAB,MAT,KOHD
CC      FORMAT (12,12X,14,1X,14,1X,15,7X,15,1X,16,1X,16)
CC      IDAYS=ICUT-IN
CC      IF (IDAYS<GE.635) IDAYS=IDAYS-635
CC      IF (IDAYS<LF.0) ERROR=ERROR+1.0
CC      PNORM=FLOAT(NORM)/FLOAT(IDAYS)
CC      PMH=FLOAT(MH)/FLOAT(IDAYS)
CC      PLAB=FLOAT(LAB)/FLOAT(IDAYS)
CC      PMAT=FLOAT(MAT)/FLOAT(IDAYS)
CC      PKOHD=FLOAT(KOHD)/FLOAT(IDAYS)
CC      JDAYS=IN-JSTART+1
CC      IF (JDAYS<GE.635) JDAYS=JDAYS-635
CC      KDAYS=IDAYS+JDAYS-1
CC
CC      BRING IN APPROPRIATE PENALTY COST
CC

```





```

DO 70 K=1,28
IF (IAC.EQ. IMATR(K)) GO TO 50
GO TO 60
50 PCOST=PMTR(K)
60 CONTINUE
70 CONTINUE
DO 1000 J=JDAYS, KDAYS, 1
ABOX(J)=ABOX(J)+PNORM
BBOX(J)=BBOX(J)+PMH
CBBOX(J)=CBBOX(J)+PLAB
DBOX(J)=DBOX(J)+PMAT
SBOX(J)=SBOX(J)+PKOHD
TBOX(J)=TBOX(J)+PCGST
EBOX(J)=EBOX(J)+1.0
1000 CONTINUE
9999 WRITE (6,9500) JSTART, JSTOP, NCARD
9500 FORMAT (11.0, //, 18X, 'SUMMARY OF DAILY STATISTICS OF NARF',
1, ' ( ENGINES ) ', //, 27X,
1, ' DATA FOR JULIAN DATES ', 14, ' TO ', 14, //, 27X,
2, ' TOTAL NUMBER OF OBSERVATIONS = ', 14, //, 7X, ' JULIAN ', 15X, ' DIRECT ',
36X, ' DIRECT ', 5X, ' DIRECT ', 20X, ' PENALTY ', 5X, ' # IN ', 7, ' 1X, ' DAY ',
44X, ' DATE ', 8X, ' NGRM ', 4X, ' MAIL HOURS ', 4X, ' LAB COST ', 3X, ' MAIL COST ',
53X, ' OVHD COST ', 8X, ' COST ', 5X, ' SHOP ', //)
K=JSTOP-JSTART
IF (K.GE.635) K=K-634
LDAYS=JSTART
DO 2000 I=1, K, 1
WRITE (6,8501) I, LDAYS, ABOX(I), BBOX(I), CBBOX(I), DBOX(I), SBOX(I),
6TBOX(I), EBOX(I)
9501 FORMAT (14, 4X, 14, 6F12.2, 5X, F4.0)
WRITE (7,8501) I, LDAYS, ABOX(I), BBOX(I), CBBOX(I), DBOX(I), SBOX(I),
8TBOX(I), EBOX(I)
8501 FORMAT (13, 1X, 14, 1X, F7.2, 1X, F7.2, 1X, F8.2, 1X, F8.2, 1X,
9F7.2, 1X, F4.0)
LDAYS=LDAYS+1
IF (LDAYS.EQ. 0366) LDAYS=1001
2000 CONTINUE
9508 WRITE (6,9508) ERROR
FORMAT (10X, 'NUMBER OF ERRORS =', F3.0)
STOP
END

```



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ABSTRACT

The objective of this study was to estimate a cost function from a Constant Elasticity of Substitution production function and a Cobb-Douglas production function for the aircraft rework and engine repair programs at the Naval Air Rework Facility, North Island, San Diego, California. The cost functions were estimated by multiple regression analysis from data aggregated from actual data taken from production records of the two programs. An attempt was made to validate the two cost functions that were obtained, and a methodology was outlined for comparing predicted costs to actual production costs at the Naval Air Rework Facility.









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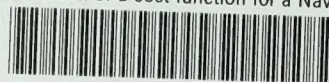
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